Standard Model particle discrete symmetry

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Abstract

By using methods of the theory of numbers, correlations have been found between integers, which are possessed of certain symmetry properties, represented in the decimal system and certain properties of elementary particles described by Standard Model. These correlations indicate a possibility of a connection existing between the symmetry of integers and the symmetry of the particles.

Keywords: Standard Model; Discrete and Finite Symmetries;

1. INTRODUCTION

It has been ten years since t-quark was experimentally discovered, an event that strengthened the belief among the physicists that the standard model (SM) is correct. The studies following this discovery enabled to define the SM parameters more accurately, while having detected no major deviations from SM [2]. According to SM, all interactions between the leptons and between the quarqs that belong to one generation are invariant relative to the gauge symmetry $SU(3)_C$ x $SU(2)_L$ x $U(1)_Y$ transformations in the the isospin space. These interactions are realized through the impact of four fundamental forces: the strong one, the electromagnetic one, the weak one and the gravitational force [3]. The gravitational force is conditioned by the space curvature and, strictly speaking, due to this fact is not to be described by SM. However, there has recently been a tendency towards a comprehensive consideration of all the four forces, based on some geometric ideas [3]. All the above-mentioned interactions are realized through an exchange of five kinds of bosons (4 vector bosons and one graviton) among three generations of fermions (3 pairs of leptons and 6 quarks), as well as among the bosons themselves.

It is well known that gauge symmetry allows to write equations which describe interactions between the particles. However this symmetry is unable to predict number of particles that take part in interaction (number of generations of fermions), numerical values of all dimensionless constants of interaction as well as some other properties of the particles. It may be supposed that these characteristics must be predicted by other, additional (non gauge) discrete symmetry, which also characterises the particles of SM.

The present work shows, by using methods usually applied in the number theory, that there is a bizarre correlation between the some fundamental particle's characteristics,

described by SM, and a certain totality of integers represented in the decimal system (base-**10**). These integers are possessed of some symmetry properties. There is a possibility that such a correlation indicates the existence of a connection between the above symmetry of particles and the symmetry of the integers represented in the decimal system.

The decimal system of numeration is an example of positional number system [4]. The novelty of the presented approach is that it includes into considera-

tion those symmetry properties of integers that are conditioned by the number system in which the integers are represented. These symmetry properties are different for different positional systems. In contrast, the number theory usually considers only common properties of integers, which don't depend of the system in which these integers are represented [5-7].
As we know, in order to write a number, the decimal system uses nine figures (that correspond to the first nine integers of the natural series) and the zero. Section 2 (below) describes symmetry test enabling to isolate the required totality of integers. In Section 3, the same test is applied to integers represented in the octal system of numeration (base-**8**) and the duodecimal system (base-**12**). It is shown that there isn't any correlation found between SM and numbers in the latter two systems.
Numbers and only numbers (no tensors, no spinors) are objects of research in the present article. We use bold type, if a written number is used as an object of research, and usual type in all other cases, for example in the case of formulas and references numeration. We use point to mark product of numbers in order to distinguish between the product of the numbers and a multidigital number (number expressed by several figures).

2. DECIMAL SYSTEM

The full above-mentioned totality of integers is isolated by means of Test ?1 that will be defined below. The first nine integers in the decimal system are one-digit numbers. Out of the nine, 5 integers are primes, the least of them being 1 (unity) which we also include into the group of primes, in contrast to the traditional number theory approach where special properties of unity are important [5-7]. We shall never use here these special properties of unity. The first correlation between SM and the said totality of integers follows immediately: the number of one-digit primes is equal to the number of boson kinds, bosons being the agents of interaction in SM.
In this case, the question arises whether there is another totality of numbers within the decimal system, whose symmetry properties are similar to those of the totality of one-digit primes? In order to answer the question, we consider some evident properties of one-digit primes.
1. The sum of digits forming each of these numbers is a prime itself. The proof: each of these numbers is formed by a single digit, hence, the sum of number-forming digits is equal to the number itself, thus being a prime by definition.
2. In case of an arbitrary permutation of the digits in a number, all the divisors of the new numbers obtained as a result of the permutation are non-iterant primes (excluding of course the unity). The proof: a given number consists of a single digit, therefore, only an identical permutation is performable, which yields the same number having just one prime divisor (excluding the unity) that is equal to the number itself, the number being a prime by definition.
3. All the digits forming the given number are different and are primes. The proof:
There are indeed no two similar digits, so as a single digit forms the number. Hence, all the digits are different and, by definition, coincide with primes. These

digits are hereafter referred to as primes.
An integer **C** is considered as conforming to Test #1 in case it is divisor of the integer, which meets all the three conditions, stated above. This is the definition of Test #1.
We proceed now to finding the totality of integers as presented in the decimal system, those meet conditions (1-3)
It is evident that such a totality cannot be found among two-digit numbers. The proof: Let us consider at the beginning a two-digit numbers composed of only odd digits. The sum of digits is even. The only even prime is **2**. Hence, the only candidate is number **11**. But number **11** is composed of two identical 1- figures, which contradicts condition (3). We consider now two-digit numbers containing figure **2**. We see that both **12** and **21** are not acceptable, since **12=2·2·3** (divisor **2** is repeated twice) which contradicts condition (2). Similarly, **23** and **32** are not acceptable, since **32= 2·2·2·2·2** (divisor **2** is repeated **5** times!). Neither are **25** and **52** suitable, since **25=5·5**. These examples exhaust all the two-digit numbers composed of unrepeated primes whose sum is itself a prime. Hence, the totality in question cannot be found among two-digit numbers.
It is evident that there are no numbers possessing properties (1-3) among the four-digit numbers. Indeed, the required four-digit number is to be composed of **4** different primes. There are only 5 digits primes in the decimal system; therefore a four-digit number ought to have either figure 2 or figure 5 in its notation, which means that one of the permutations will yield a number ending with either 2 or 5. Such numbers are not acceptable for our purpose, since at least six of them obtained as a result of a permutation are divided into by either **2** or **5**, i.e. they contain at least six times either divisor **2** or divisor 5, which is in conflict with condition (2). For the same reason, three-digit numbers containing either figure **2** or figure **5** also are not acceptable.
It becomes clear now that, out of the infinity of multidigital numbers, only six numbers conform to conditions (1-3). Namely, the following numbers obtained as a result of permutation of digits composing the number C

C = 137 (1)

137, 317, 731, 371, 173, 713 (2)

It can be easily established that numbers **137, 173** and **317** are primes, while numbers **371, 713 & 731** are composed ones:

371 = 7·53, 713 = 23·31, 731 = 43·17 (3)

All the factors in (3) are unrepeated primes, with the exception, of course, of unity 1 that is a divisor for any number. Hence, the 6 integers (**2**) obtained as a result of permutation of the digits composing number **137** is the only totality of three-digit numbers in the decimal system that meets the conditions of Test #1.
This analysis indicates that the totality of one-digit primes in the decimal system can be complemented with the six three-digit integers (2) that realize an irreducible representation of the group of permutations involving six elements. These three-digit integers led us to six other integers, three pairs of products of primes (3). Thus, we have arrived at the second correlation between a number and SM, since Test #1 has isolated, out of the natural series, 12 different in-

tegers connected with the three-digit integers, 12 being also the number of the known fundamental fermions in SM.
The third correlation is found due to the fact that we have obtained 6 three-digit integers that may correspond to the six quarks characterized by the symmetry SU (3)C, as well as three pairs of primes that may correspond to the three lepton doublets represented by the $SU(2)_L$ symmetry. It is naturally to suppose that every pair of factors in (3) corresponds to pair of leptons that belong to one generation. In SM, the quarks are also represented by doublets in the $SU(2)_L$ symmetry. At the same time, each individual quark is also represented by a $U(1)_Y$ singlet, i.e. by the symmetry of a scalar within the space of the weak isospin, which makes it possible for a quark to be represented by a separate number. Three neutrinos among the leptons are not represented by singlets $U(1)_Y$ in Lagrangian, therefore, all the leptons can be represented only by three pairs of integers that correspond to the three $SU(2)_L$ doublets (see Table 1 in [2]).
Let us now consider in more details equalities (3). We can see that among the three pairs of prime factors in (3) there is an one strongly selected pair. We bear in mind the pair $\mathbf{7} \cdot \mathbf{53}$. Only this pair contains one-digit prime. The presence of selected pair of integers in (3) is the fourth correlation between the integers and the CM particle properties, so as among the three pairs of leptons also exists an one strongly selected pair, namely, the first generation pair of leptons, the pair electron e and electron's neutrino ne. This pair is selected for the reason that only this pair contains stable electrically charged lepton with the minimum mass, stable as well as stable photon, which is the only experimentally observable stable vector boson.
It follows from our consideration that it is impossibly to realize one to one correspondences between the particles and integers in the frames described above correlation between the particles and said totality of integers. Integer 7 must correspond to two particles one boson and one lepton. It is naturally to suppose that these particles are photon and electron possessing common property stability.
Now we shall proceed further description of correspondences between the integers and the SM particles. We can notice that every product in the (3) content two different kinds of factors. In the factors of first kind sum of digits is odd and in the factors of second kind sum of digits is even. In order to have maximum generality we must conclude (taking in account that sum of the digits in the every three-digits integer in (2) is odd) that factors of first kind in (3) correspond to electrically charged leptons and the factors of second kind correspond to uncharged leptons or neutrinos. This follows from the fact that according to our previously made supposition three-digit integers in (2) correspond to quarks, which are electrically charged particles. So we shall have that every integer corresponding to electrically charged fermion will have odd sum of digits. From the facts that only one electrically charged boson is observable in SM, namely Z-Boson, and there is only one even one-digit prime we must conclude that in the case of bosons, on the contrary, integers corresponding to electrically charged particle must have even sum of the digits.

For the identification of all fermions that belong to one generation we shall do next. It is naturally to suppose that three-digit integers being in the left parts of equalities (3) correspond to first quarks that belong to the same generation that the pairs of leptons presented by factors in the right parts of equalities (3). The second quark of the generation may be found by next way. We can notice that only every middle digit in the all of three-digit integers in (3) distinguishes one from other. This means that the middle digit may be used for identification of the generation. The integer that corresponds to the second quark of generation must have the same middle digit as the middle digit in the integer corresponding to the first quark. This integer exists and only, so as position of every digit repeated only twice in the integers corresponding to quarks in accordance with (2). So we have realized one to one correspondence between the fermions of one generation and the integers.

Summing up all above results we must specially notice the next fact. In spite of the quantity of suppositions that made above, nevertheless, it is extraordinary that it is successful to achieve, free from contradictions, the above described numerous correlations between the totality of integers, complying with the Test #1 in the decimal system, and SM particle properties. In other to emphasize the improbability of all these coincidences below, for example, presented totalities of integers complying with the test ?1 in the some other systems of numeration.

3. OCTAL AND DUODECIMAL SYSTEMS

In this section, Test #1 is applied to the octal (base-**8**) and the duodecimal (base-**12**) systems, which represent integers **1,3,7** by means of a single digit, i.e. these integers are one-digit numbers within said systems.

In order to avoid misunderstanding, we will mark by **8** in the bottom right corner all the numbers represented in the octal system, and with **12** all the numbers represented in the duodecimal system. As to numbers represented in the decimal system, they will continue to carry no marking. E.g., integer 12 is presented as $\mathbf{10_{12}}$ in the duodecimal system and as $\mathbf{14_8}$ in the octal system. New figures corresponding to integers **10** and **11** as presented in the duodecimal system, are noted as $\underline{\mathbf{10}}$ and $\underline{\mathbf{11}}$, respectively.

In the octal system, the nonary system (base-**9**) and the undenary (base-**11**) system there is the same number of one-digit primes as in the decimal system. In the duodecimal system, an additional one-digit prime is formed presented by the figure $\underline{\mathbf{11}}$. This immediately break the correspondence between the number of bosons serving as interaction agents in SM, and the number of one-digit primes. In view of these considerations, we now proceed to the diccussion of Test #1.

In the octal system, two totalities of two-digit integers conform with Test #1: $\mathbf{128 = 2 \cdot 5}$, $\mathbf{21_8} = \mathbf{17}$ and $\mathbf{23_8} = \mathbf{19,\ 32_8 = 13 \cdot 2}$. In the duodecimal system also two totalities of two-digit integers meets the conditions of Test #1**:** $\mathbf{2\underline{11}_{12} = 17}$ **,** $\mathbf{\underline{11}2_{12} = 2 \cdot 67}$ and $\mathbf{25_{12} = 29}$ and $\mathbf{52_{12} = 31 \cdot 2}$. Integers $\mathbf{137_8}$ and $\mathbf{137_{12}}$ don't conform to the Test #1, so as $\mathbf{173_8 = 123,\ 713_8 = 459,\ 173_{12} = 231,\ 713_{12} = 1023}$, all four integers have common divisor **3**. The number C = **137** itself is presented as $\mathbf{211_8}$ in the octonal system and as $\mathbf{\underline{11}5_{12}}$ in the duadecimal system. In both cases, this number does not confrom with Test #1. In the first case, condition (3) is violated, and in the second case condition (1)

is violated (see the previous section). However, the integer **137** complies with Test #1 for example in the sexagecimal system (base-**60**) being represented in the form **$137=2\underline{17}_{60}$, $\underline{17}2_{60}=1022=2\cdot 7\cdot 7\cdot .3$.**
The examples given above show that neither the octal numeration nor the the duodecimal one bear any, even remote, correlations resembling those multiple correlations that were found between SM and the integers presented in the decimal numeration. Similarly, no correlations are found between SM and the integers in the nonary or the undenary systems, which can be easily proved by implementing Test #1
The main result obtained in the present work may be summarized by the next conclusion. The presence of all above-described multiple correlations between the characteristics of the SM particles and integers complying with the test #1 in one of the systems of numeration may be a result of an inconceivable accidental coincidence. The second, and in the author's vew more probable variant, these correlations are a manifestation of laws of symmetry that are obeyed both by the integers presented in this system of numeration and the fundamental particles in SM. If the latter is true, this symmetry, in the case of particles, reflects inner structure of the standard model particles, just as natural series symmetry of atomic nucleous charges reflects an inner structure of the nucleai. This inner structure of the particles must be observable in experiment at the distances $r < r_0$. Where $r_0 \sim 10^{-17}$ cm - is minimum distance of interaction available today in the collider experiments.

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